Multi–soliton Solutions of Two–dimensional Matrix Davey–Stewartson Equation

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Abstract

The explicit formulae for m–soliton solutions of (1+2)–dimensional matrix Davey–Stewartson equation are represented. They are found by means of known general solution of the matrix Toda chain with the fixed ends [1]. These solutions are expressed trough $m+m$ independent solutions of a pair of linear Shrödinger equations with Hermitian potentials.

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1 Davey–Stewartson Equation

Let u, v are two non–singular $s \times s$ matrix functions of x, y, i.e. each matrix element is a function of x, y coordinates of two–dimensional space. Partial derivatives of these functions up to some sufficient large order are assumed to exist.

We define the matrix Davey–Stewartson equation (DSE) as the following partial differential equation.

$$iu_t + au_{xx} + bu_{yy} - 2au \int dy(u^*u)_x - 2b \int dx(uu^*)_y \cdot u = 0 \tag{1}$$

where a, b are arbitrary real numbers and z^* is a Hermitian conjugate of a matrix z. It will be convenient to deal not with the equation (1), but with the following expanded system, which we call the matrix Davey–Stewartson system (DSS).

$$\begin{aligned} iu_t + au_{xx} + bu_{yy} - 2au\int dy(vu)_x - 2b\int dx(uv)_y \cdot u = 0 \\ -iv_t + av_{xx} + bv_{yy} - 2a\int dy(vu)_x \cdot v - 2bv\int dx(uv)_y = 0 \end{aligned} \tag{2}$$

Bellow for definity we had chosen $a = b = 1$. It is easy to see that the DSE is the system (2) under additional condition $v = u^*$. This condition we call the condition of reality.

In the case $s = 1$ (scalar case), when u, v are scalar functions and the order of the multipliers is not essential (1) is the usual, well–known Davey–Stewartson equation [2]. In the scalar case soliton solutions of the DSS were obtained in [3].

2 Discrete substitution

The method we use to solve the problem is based on the discrete transformation investigation. Here we consider concrete discrete transformation, which is important for our problem.

By direct calculations can be checked that (2) is invariant with respect to the following change of unknown matrices u, v.

$$\tilde{u} = v^{-1}, \quad \tilde{v} = \left[vu - (v_x v^{-1})_y\right] v \equiv v \left[uv - (v^{-1} v_y)_x\right] \tag{3}$$

Here $\tilde{u}, \tilde{v}$ denote the "new", transformed operators. Invariance means that matrices $\tilde{u}, \tilde{v}$ satisfy exactly the same system (2), which matrices u, v satisfy. Mapping (3) is an invertible one and the "old" matrices u, v can be expressed trough the "new" ones.

$$v = \tilde{u}^{-1}, \quad u = \left[\tilde{u}\tilde{v} - (\tilde{u}_y \tilde{u}^{-1})_x\right] \tilde{u} \equiv \tilde{u} \left[\tilde{v}\tilde{u} - (\tilde{u}^{-1}\tilde{u}_x)_y\right] \tag{4}$$

Transformation (3) can be rewritten in the form of an infinite chain of equations in two equivalent ways as

$$\left((v_n)_x v_n^{-1}\right)_y = v_n v_{n-1}^{-1} - v_{n+1} v_n^{-1}, \quad u_{n+1} = v_n^{-1} \tag{5}$$

or as

$$\left(v_n^{-1}(v_n)_y\right)_x = v_{n-1}^{-1} v_n - v_n^{-1} v_{n+1} \tag{6}$$

where (v_n, u_n) is a result of n–times substitution (3) applied to some initial matrices v_0, u_0. Sequences (5,6) with $v_{-1}^{-1} = v_N = 0$ boundary conditions we call the matrix Toda chain with the fixed ends.

In the scalar case $s = 1$ general solution of the Toda chain with the fixed ends was found in [4] for all series of semi–simple algebras, except E_7, E_8. In [5] this result was reproduced in terms of invariant root technique applicalible to all semi–simple series.

The explicit general solution of the matrix Toda chain with the fixed ends was found in [1]. It was expressed trough $N+N$ arbitrary independent matrix functions of a single argument $X_r(x), Y_r(y)$ as

$$v_0 = \sum_{r=1}^{N} X_r Y_r \tag{7}$$

To (7) correspond the following formula for u_N.

$$u_N = \sum_{r=1}^{N} \tilde{Y}_r(x)\tilde{X}_r(y) \tag{8}$$

Matrices $\tilde{X}, \tilde{Y}$ here are not arbitrary ones, but in some way depend from X, Y. Both these results (7) and (8) will be used in further consideration.

3 General Strategic

We are going to solve the DSS (2) under condition of reality $u = v^*$. Here we describe how the discrete transformation is used for that. General idea is the following. At first we take some obvious solution of the DSS (2). It may be not a solution of the problem (reality condition may be not satisfied). Then, by means of the discrete transformation (3), we get from that initial, obvious solution a solution which is satisfy to the condition of reality.

For $u_0 = 0$ the first equation of the system (2) is satisfied identialy, the second one gives

$$-iv_{0t} + v_{0xx} + v_{0yy} + V_1(t,x)v_0 + v_0V_2(t,y) = 0 \tag{9}$$

where V_1, V_2 are arbitrary $s \times s$ matrix functions of their arguments (these terms arise from the undefined integrals $\int dx(uv)_y$, $\int dy(uv)_x$ in the system (2)). Obviously, condition of reality is not satisfied for this solution. But after enough times discrete transformations (3) it is possible to come to the solution for which it is satisfied. To clarify this, let us consider some solution u, v for which condition of reality is satisfied, $u = v^*$. Denoting u_1, v_1 and u_{-1}, v_{-1} the results of direct (3) and inverse (4) substitutions respectively, it can be easily proved that $u_{-1} = {v_1}^*$ and $v_{-1} = u_1^*$. On the m-th step, we have: $u_{-m} = {v_m}^*$ and $v_{-m} = u_m^*$, where index m $(-m)$ correspond to the

result of the m-times direct (inverse) transformation. And vice versa one can prove that if we begin from solution $u_0 = 0, v_0$ and after 2m–times discrete transformation receive $u_{2m} = v_0^*, v_{2m} = 0$, the solution in the middle of the chain automatically satisfy the reality condition, $u_{m+1} = v_{m+1}^*$.

The system arising from equations $u_0 = v_{2m} = 0$ is already resolved by the formula (7). So it remains to solve the equation: $u_{2m} = v_0^*$. It leads to the following relations between $X_r, \tilde{X}_{\sigma[r]}$ and $Y_r, \tilde{Y}_{\sigma[r]}$

$$X_r^* = \tilde{X}_{\sigma[r]} \qquad Y_r^* = Y_{\sigma[r]} \tag{10}$$

where σ denotes one of the $(2m)!$ possible permutations of the $2m$ low indexes. To solve (10) at first it is necessary to find the dependence of $\tilde{X}$ and $\tilde{Y}$ from X and Y respectively. Finally equation (9) in terms of X_r, Y_r can be rewritten as

$$-iX_{rt} + X_{rxx} + V_1(t,x)X_r = 0 \quad -iY_{rt} + Y_{rxx} + Y_r V_2(t,y) = 0 \tag{11}$$

Thus to find the m–soliton solutions of the DSE (1) it is necessary to undertake the following steps

- find the dependence $\tilde{X}_i\,(X_1, \dots, X_{2m})$ and $\tilde{Y}_i\,(Y_1, \dots Y_{2m})$
- solve the system (10)
- find such a dependence of matrix functions X_r, Y_r from the time argument, which will satisfy to the system (11).

After this substituting X_r, Y_r in (7) we find v_0, for which $u_{m+1} = v_{m+1}^*$ is some partial (m–soliton) solution of the Davey–Stewartson equation (1).

4 Scalar Case

To gain some experience firstly we consider the scalar case $s = 1$, for which much of the necessary calculation steps are well–known and much simpler then in the general matrix case.

In this case for the mentioned above boundary conditions the following formulae for arbitrary k takes place [7].

$$u_k = \frac{Det_{k-1}}{Det_k} \quad v_k = \frac{Det_{k+1}}{Det_k}, \quad Det_{-1} \equiv 0, \quad Det_0 \equiv 1 \tag{12}$$

where Det_k is the principle minor of dimension k of the matrix ($v^0 \equiv v_0$)

$$\begin{pmatrix} v^0 & v^0_x & v^0_{xx} & .... \\ v^0_y & v^0_{xy} & v^0_{xxy} & ..... \\ v^0_{yy} & v^0_{xyy} & v^0_{xxyy} & ..... \\ ... & .... & .... & ..... \\ ... & .... & .... & ..... \end{pmatrix}$$

and v^0 is determined by (7), where X_r, Y_r are arbitrary scalar functions of their arguments. Substituting (7) in the formula for u_{2m} from (12) and comparing with (8) we find

$$\tilde{X}_r(x) = \frac{W_{2m-1}(X_1, X_2, ..., X_{r-1}, X_{r+1}, ...X_{2m})}{W_{2m}(X_1, X_2, ...X_{2m})} \tag{13}$$

Here and bellow W_k denotes a Wrosnkian of dimension k constructed from the functions in the brackets.

$$W_k(g_1, \dots, g_k) \equiv \begin{vmatrix} g_1 & g_2 & \cdots & g_k \\ g_1' & g_2' & \cdots & g_k' \\ \vdots & \vdots & \ddots & \vdots \\ g_1^{(k-1)} & g_2^{(k-1)} & \cdots & g_k^{(k-1)} \end{vmatrix} \qquad W_0 \equiv 1 \tag{14}$$

Expressions for $\tilde{Y}_r$ can be received from (13) by the simple exchange $X \to Y$.

In the condition of reality (10) we use the permutation $\sigma[r] = 2m - r + 1$ To resolve (10) and (11) it is suitable to represent the functions X_r, Y_r in the Frobeniouse–like form

$$\begin{aligned} &X_1 = \phi_1, \quad X_r = \phi_1 \int dx \phi_2 \dots \int dx \phi_r \\ &Y_1 = \psi_1, \quad Y_r = \psi_1 \int dx \psi_2 \dots \int dx \psi_r \end{aligned} \tag{15}$$

From (13) we find

$$\tilde{X}_{2m} = \Big(\prod_{k=1}^{2m} \phi_k\Big)^{-1} \quad \tilde{X}_r = \Big(\prod_{k=1}^{2m} \phi_k\Big)^{-1} \int dx \phi_{2m} \dots \int dx \phi_{2m-r} \tag{16}$$

After this the reality condition (10) takes the following form

$$\phi^*_r = \phi_{2m-r+2} \quad (r = 2, 3, ...2m), \quad \phi_{m+1} = \phi^*_{m+1} = \Big(\prod_{k=1}^{m} \phi_k \phi^*_k\Big)^{-1} \tag{17}$$

And from (11) we have

$$\phi_{rt} = \left(\phi_r \left(\ln \phi_r \prod_{k=1}^{r-1} \phi_k^2\right)'\right)' \tag{18}$$

The imaginary unity i here is included into the time variable, which therefore should be treated as a pure imaginary from this moment. Independently can be checked that the systems (17) and (18) are compatible and if (18) is obeyed for some $\phi_r, \quad r \leq m$, for ϕ_{2m-r+2} it also holds. Therefore it is enough to consider only equations with $r \leq m$ in the system (18). Now we introduce the new unknown functions $f_r^{-1} = \phi_1 \cdots \phi_r, \quad r \leq m+1$. From (18) we find

$$(f_r^{-1} f_{r-1})_t = -\left(f_r^{-1} f_{r-1} (\ln f_r f_{r-1})'\right)' \tag{19}$$

From (17) it follows that $f_m^* = f_{m+1}^{-1}$. Substituting this in the $(m+1)$-th equation of the last system, we have

$$(f_m f_m^*)_t = \left(f_m f_m^* \left(\ln f_m f_{m-1}^*\right)'\right)' \tag{20}$$

Equation (20) is equivalent to one–dimensional Shrödinger equation with arbitrary real potential.

$$f_{mt} + f_m'' = U f_m, \quad U = U^* \tag{21}$$

Now let us consider the m–th equation of the system (19).

$$(f_m^{-1} f_{m-1})_t = -\left(f_m^{-1} f_{m-1} (\ln f_m f_{m-1})'\right)' \tag{22}$$

Partially resolving it as

$$f_m^{-1} f_{m-1} = z', \quad (\ln f_m f_{m-1})' = -\frac{z_t}{z'} \tag{23}$$

and excluding function f_{m-1}, we conclude that function $z f_m$ satisfy exactly the same equation (21), which f_m satisfy. Denoting $u_i, \quad (1 \leq i \leq m)$ m independent solutions of (21) we find

$$f_m = u_1, \quad z f_m = u_2 \Longrightarrow f_{m-1} = z' f_m = \frac{\begin{vmatrix} u_1 & u_2 \\ u_1' & u_2' \end{vmatrix}}{u_1} \tag{24}$$

In the general case, for arbitrary i the following formula holds.

$$f_r = \frac{W_{m-r+1}}{W_{m-r}} \quad r \leq m \tag{25}$$

where $W_i = W_i(u_1, \ldots, u_i)$.

To prove (25) we use the well–known Jacobi identity for determinants. Let T is some infinite in both directions matrix; $D_n(T)$ denotes the determinant of its $n \times n$ principle minor, T^s is the matrix received from T by deleting its s-th column and T_p–by deleting its p-th row. In this notations Jacobi identity takes the form

$$D_n(T)D_n(T_n^n) - D_n(T^n)D_n(T_n) = D_{n+1}(T)D_{n-1}(T) \tag{26}$$

From (26) the following identity can be easily derived.

$$W_i\overline{W}_i' - W_i'\overline{W}_i = W_{i-1}W_{i+1} \tag{27}$$

where $\overline{W}_i = W_i(u_i \to u_{i+1}) = W_i(u_1, u_2, \ldots, u_{i-1}, u_{i+1})$.

Now let us partially resolve equation (19) for arbitrary r.

$$f_{r-1} = z' f_r \qquad (\ln f_r f_{r-1})' = -\frac{z_t}{z'} \tag{28}$$

Excluding f_{r-1} from the last system we find that f_r and zf_r are different solutions of the same equation. And if f_r is given by the formula (25), zf_r can be determined as

$$zf_r = \frac{\overline{W}_{m-r+1}}{W_{m-r}}$$

After this from (28) with the help of the identity (27) we easily find

$$f_{r-1} = \frac{W_{m-r+2}}{W_{m-r+1}}$$

Thus formula (25) is proved by induction.

Finally for functions ϕ_r from the definition of f_r and formulae (17) we have

$$\begin{gathered} \phi_{m+1} = v_1 v_1^* \quad \phi_r = \frac{W_{m-r+2}W_{m-r}}{W_{m-r+1}^2} \\ \phi_1 = \frac{W_{m-1}}{W_m} \qquad \phi_r^* = \phi_{2m-r+2} \qquad r \leq m \end{gathered} \tag{29}$$

The analogues expressions take place for functions ψ_k

$$\psi_{m+1} = v_1 v_1^* \quad \psi_r = \frac{W_{m-r+2} W_{m-r}}{W^2_{m-r+1}}$$
$$\psi_1 = \frac{W_{m-1}}{W_m} \qquad \psi_r^* = \psi_{2m-r+2} \qquad r \leq m \tag{30}$$

In (30) $W_i = W_i(v_1, \ldots, v_i)$ and $v_i \equiv v_i(y) \quad 1 \leq i \leq m$ are m independent solutions of $(1+1)$–dimensional linear Shrödinger equation with some arbitrary real potential V.

$$v_{it} + v_i'' = V v_i \quad V = V^* \tag{31}$$

5 Matrix case

Here we consider a general problem as it was formulated in the sections 1 and 3. We find m–soliton solutions of the DSE for an arbitrary dimension of the unknown matrix u. We therefore receive matrix generalizations of all formulae of the previous section. It turns out that quasi–determinants of matrices with non–commutative entries play the role of usual determinants. Conception of a quasi–determinant was introduced recently by Gelfand and Retarh [9]. We use an independent technique, more appropriate for our particular case, but quasi–determinants can be used as well.

With the chain (5,6) under mentioned above boundary conditions we connect the following recurrent relations

$$R_n \equiv v_n^{-1} v_{ny}, \quad S_n^q \equiv \sum_{k=0}^{n-1} (S_{k_y}^{q-1} + R_k S_k^{q-1}) \tag{32}$$

with the boundary conditions $S_i^0 \equiv 1$ for arbitrary i. From definitions (32) and equations (5,6) we easily find

$$S_n^1 = \sum_{k=0}^{n} R_n \quad S_0^q = v_0^{-1} v_0 \underbrace{y \cdots y}_{q} \tag{33}$$

$$v_{n+1} = -v_n (S_{n+1}^1)_x = (-1)^{n+1} v_0 (S_1^1)_x (S_2^1)_x \cdots (S_{n+1}^1)_x \tag{34}$$

For such introduced (32) matrix functions S_n^q the following relation is true.

$$S_n^q = \left[(S_{n-1}^1)_x\right]^{-1}(S_{n-1}^{q+1})_x \tag{35}$$

Now let us find the dependence of $\tilde{X}$ from X. For that we use the fact that each matrix function X_i is determined only by matrices $X_1,\ldots,X_{2m}$ and therefore we can choose matrices $Y_1,\ldots,Y_{2m}$ in an arbitrary way. It is convenient to choose

$$Y_i = \frac{y^{i-1}}{(i-1)!}E \quad v_0 = X_1 + \frac{y}{1!}X_2 + \cdots + \frac{y^{2m-1}}{(2m-1)!}X_{2m} \tag{36}$$

where E is a unity $s\times s$ matrix. Substituting (36) in the expression for v_{2m-1} from (34) we find

$$v_{2m-1}^{-1}\Big|_{y=0} = -\left[X_1(T_0^1)_x(T_1^1)_x\cdots(T_{2m-2}^1)_x\right]^{-1}, \tag{37}$$

where matrices T_q^n are determined by the following relations

$$T_n^q = \left[(T_{n-1}^1)_x\right]^{-1}(T_{n-1}^{q+1})_x \tag{38}$$

with the boundary conditions

$$T_0^q = S_0^q|_{y=0} = X_1^{-1}X_{q+1} \tag{39}$$

Expression (37) correspond to one of the functions $\tilde{X}_i$. And because these functions can be enumerated in various ways we can choose

$$(\tilde{X}_1)^{-1} = X_1(T_0^1)_x(T_1^1)_x\cdots(T_{2m-2}^1)_x \equiv F(x_1,\ldots,x_{2m}) \tag{40}$$

Formula for arbitrary i can be received from (40) by the cycler permutation of the indexes.

$$\tilde{X}_i^{-1} = (-1)^{i-1}F(\sigma_i[x_1,\ldots,x_{2m}]) \tag{41}$$

Arbitrary multiplier can be added in the formula (41). It will be counted in the expression for $\tilde{Y}_i$. We added $(-1)^{i-1}$ to do further calculations more convenient. After this using (41) and (34) we find

$$\tilde{Y}_1^{-1} = -(Q_{2m-2}^1)_y\cdots(Q_0^1)_y \quad Y_1 \equiv G(Y_1,\ldots,Y_{2m}) \tag{42}$$

$$\tilde{Y}_i^{-1} = (-1)^i G(\sigma_i[Y_1, \ldots, Y_{2m}]) \tag{43}$$

where

$$Q_n^s = (Q_{n-1}^{s+1})_y \left[(Q_{n-1}^1)_y\right]^{-1} \quad Q_0^s = Y_{s+1} Y_1^{-1} \tag{44}$$

Now as in the previous section we represent the initial functions X_r, Y_r in the Frobenious–like form.

$$\begin{aligned} &X_1 = \phi_1, \quad X_2 = \phi_1 \int dx \phi_2, \quad X_3 = \phi_1 \int dx \phi_2 \int dx \phi_3, \quad \ldots \\ &Y_1 = \psi_1, \quad Y_2 = \int dx \psi_2 \cdot \psi_1, \quad Y_3 = \int dx \left(\int dx \psi_3 \cdot \psi_2 \right) \cdot \psi_1, \quad \ldots \end{aligned} \tag{45}$$

After permutation of the indexes the formulae for $\tilde{X}_r$ will coincide with (16). The only difference is that in the matrix case the order of the multipliers must be taken into account.

$$\begin{aligned} &\tilde{X}_1 = p, \quad \tilde{X}_2 = \int dx \phi_{2m} \cdot p, \quad \tilde{X}_3 = \int dx \left(\int dx \phi_{2m-1} \cdot \phi_{2m} \right) \cdot p, \quad \ldots \\ &\tilde{Y}_1 = -s, \quad \tilde{Y}_2 = -s \int dx \psi_{2m}, \quad \tilde{Y}_3 = -s \int dx \psi_{2m} \int dx \psi_{2m-1}, \quad \ldots \end{aligned} \tag{46}$$

where $p = (\phi_1 \cdots \phi_{2m})^{-1}$ and $s = (\psi_{2m} \cdots \phi_1)^{-1}$. The condition of reality taken in the form $\tilde{X}_r = X_r^*, \quad \tilde{Y}_r = Y_r^*$ gives

$$\begin{aligned} &\phi_r^* = \phi_{2m-r+2} \quad 2 \le r \le m, \\ &\phi_{m+1}^{-1} = (\phi_{m+1}^*)^{-1} = (\phi_1 \phi_2 \cdots \phi_m)^* (\phi_1 \phi_2 \cdots \phi_m) \\ &\psi_r^* = \psi_{2m-r+2} \quad 2 \le r \le m, \\ &\psi_{m+1}^{-1} = (\psi_{m+1}^*)^{-1} = -(\psi_m \psi_{m-1} \cdots \psi_1)^* (\psi_m \psi_{m-1} \ldots \psi_1) \end{aligned} \tag{47}$$

The fact that all functions X_r are solutions of the same equation (11) leads to the following system.

$$-(\phi_s)_t + (2(\phi_1 \phi_2 ... \phi_{s-1})^{-1} (\phi_1 \phi_2 ... \phi_{s-1})' \phi_s + \phi_s')' = 0 \tag{48}$$

Introducing new the functions $f_r^{-1} = \phi_1 \phi_2 \ldots \phi_r$ from (48) we find

$$-(f_{r-1} f_r^{-1})_t = \left[f_{r-1}' f_r^{-1} - f_{r-1} (f_r^{-1})' \right]' \tag{49}$$

Then from (47) and (49) we conclude that f_m is a solution of $(1+1)$–dimensional linear Shrödinger equation with Hermitian potential.

$$f_{mt} + f_m'' = W f_m, \quad W = W^* \tag{50}$$

Solution of the system (49) can be found by the same scheme as in the previous section. Matrix case does not require to use the Jacobi identity, because recurrent definitions are used.

$$f_1 = u_1 \quad f_{m-r} = (U_r^1)' \cdots (U_0^1)' u_1 \tag{51}$$

Matrix functions U_n^q are determined by the following recurrent relations.

$$U_n^q = (U_{n-1}^{q+1})' \left[(U_{n-1}^1)' \right]^{-1}$$

with the boundary conditions

$$U_0^r = u_{r+1} u_1^{-1}$$

where matrices u_r are different solutions of the equation (50). From this we find the formulae for ϕ_{m-r} for arbitrary r. Finally we have

$$\begin{aligned} &\phi_1 = f_1^{-1} \qquad \phi_{m-r} = (U_r^1)' \quad 0 \le r \le m-2 \\ &\phi_{m+1} = u_1 u_1^* \quad \phi_r^* = \phi_{2m-r+2} \quad 2 \le r \le m \end{aligned} \tag{52}$$

For ψ_i we find

$$\begin{aligned} &\psi_1^{-1} = v_1 (V_{m-2}^1)' \cdots (V_0^1)' v_1 \\ &\psi_{m-r} = (U_r^1)' \quad 0 \le r \le m-2 \\ &\psi_{m+1} = -v_1 v_1^* \quad \psi_r^* = \psi_{2m-r+2} \quad 2 \le r \le m \end{aligned} \tag{53}$$

where

$$V_n^q = \left[(V_{n-1}^1)' \right]^{-1} (V_{n-1}^{q+1})' \quad V_0^r = v_1^{-1} v_{r+1}$$

and matrices $v_i(y)$ are different solutions of $(1+1)$–dimensional linear Shrödinger equation with arbitrary Hermitian potential.

$$v_{it} + v_i'' = v_i M, \quad M = M^*$$

Now substituting (52,53) directly in (45) and (7) and then in the formula for v_{m+1} from (34) we find the m–soliton solution of the matrix DSE. We do not write down the corresponding expression, because it can be easily received, but is too large to represent it here.

6 The Simplest Example of One–Soliton Solution

Substituting $m = 1$ in the formulae of the last section we find

$$v_0 = X_1 Y_1 + X_2 Y_2, \quad X_1 = \phi_1, \quad X_2 = \phi_1 \int dx \phi_2$$

$$Y_1 = \psi_1, \quad Y_2 = \int dx \psi_2 \cdot \psi_1$$

After this we find the following expression for the one–soliton solution of the matrix DSE.

$$u_1 = \psi_1^{-1} \left(1 + \int dx \phi_2 \int dy \psi_2 \right)^{-1} \phi_1^{-1} \tag{54}$$

Matrix functions $\phi(t,x), \psi(t,y)$ are determined by u, v solutions of the one–dimensional linear Schrödinger equations.

$$\phi_1 = u^{-1}, \quad \phi_2 = uu^*$$

$$\psi_1 = v^{-1}, \quad \psi_2 = -v^* v$$

$$u_t + u_{xx} + uM_1(t,x) = 0$$

$$v_t + v_{yy} + M_2(t,y)v = 0, \quad M_{1,2} = M^*_{1,2}$$

7 Conclusion

The main result of the paper is the explicit expressions for the m–soliton solutions of the (1+2)–dimensional matrix Davey–Stewartson equation. By means of the corresponding formulae of the sections 4 and 5 these solutions are expressed trough $m + m$ independent solutions of a pair of linear (1+1)–dimensional Shrödinger equations.

From the group–theoretical point of view it means that we had realized the finite–dimensional representation of the group of integrable mappings. This viewpoint remained beyond our concrete calculations.

Note that restriction with the finite–dimensional matrices is absolutely nonessential. We had never used this restriction and moreover the dimension (s) was not included in any expression.

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